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JIGSAW PUZZLE SOLVER USING SHAPE AND COLOR

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ABSTRACT

The jigsaw puzzle assembly problem is significant in that it can be applied to diverse areas such as repair of broken objects, restoration of archaeological findings, molecular docking problem for drug design, etc. This paper describes a new *pictorial* jigsaw puzzle solver which, in contrast to previous *apictorial* jigsaw puzzle solvers, uses chromatic information as well as geometric shape. We develop three new puzzle assembly algorithms(TSP&Kbest-based, TSP&AP-based, and AP-based algorithm) and new boundary and color matching operation.

We tested the new puzzle solver with 6 different sets of color puzzle pieces. Experimental results show that chromatic information greatly aids in seeking the solution to the jigsaw puzzle problem. It is also discovered that in terms of how rapidly each assembly algorithm reaches a solution, the TSP&Kbest-based algorithm is the best, followed by TSP&AP-based algorithm, and followed by AP-based algorithm.

1. INTRODUCTION

The *partial boundary matching* operation is one of the operations widely used in applications such as shape fitting [5, 10, 1] and object recognition [13, 8], where the entire boundary may not be available. The 2D jigsaw puzzle, which is well known to be an instance of the general *partial boundary matching* problem, has long been a challenging problem in pattern recognition and computer vision field, since its computerized solution was first introduced by Freeman [5]. The jigsaw puzzle problem is significant since it contains many popular problems encountered in image processing applications and has a great potential for interdisciplinary research such as molecular docking problem [7, 9] and computer assisted anthropology system [6]. Finding the solution to the jigsaw puzzle assembly is closely related with *square-tiling problem* and is proven to be an NP-complete problem [1].

Much work [5, 10, 1, 15, 16, 11] has been done on solving the jigsaw puzzle problem since the early 1960's. However, earlier approaches are principally concerned with the geometric shape information of the puzzle pieces. None of the previous approaches considers other useful information such as colors, surface markings, or textures as additional cues to solving the jigsaw puzzle problem.

One way to develop a more robust solution to the jigsaw puzzle problem is to seek a more complete and efficient use of the available information. Motivated by this fact, this paper proposes a new pictorial jigsaw puzzle solver, which uses chromatic information as well as partial boundary information. We have developed three new puzzle assembly algorithms and new boundary and color matching operation.

2. NEW APPROACH

The new pictorial jigsaw puzzle solver is broken down into four distinct steps: *preprocessing, local matching, global solution*, and *solution display* steps. The preprocessing step deals with acquiring images, segmenting images, extracting salient features for later processing, etc. The local matching step makes use of new boundary and color matching operation to compute local matching scores between every pair of partial boundaries. In assembling a large puzzle with pieces having similar shape, the matching scores solely based on local shape and color information might lead to an incorrect solution, because the errors introduced by cameras, feature extractions, and numerical precision of computation may let a similar but wrong piece obtain the best matching score. The global solution step uses three new assembly algorithms to delete such local distortions and thus seek a globally correct solution. Finally, in the solution display step, puzzle pieces are fitted together in accordance with the obtained global solution.

2.1. Preprocessing Stage

We assume that puzzle pieces are placed on a background with a uniform color. The NEC color camera equipped with a Computar 6.0 mm lens is right above the puzzle pieces and fixed at a same height until all the pieces in a puzzle are captured. The camera interface uses the camera calibration algorithm developed by Stevenson and Fleck [14] to remove perspective image distortions. Each image(213 $\times$ 293) contains one puzzle piece. Images are represented by the 8-bit log-opponent color model.

The basic idea of detecting the outer boundary of a color puzzle piece is that an initial image is segmented into two distinct regions (a background region and a puzzle piece region) by *histogram thresholding*. For image segmentation, four features are taken into account: red/green, blue/yellow, hue, and saturation in log-opponent color representation. The binary segmented image is smoothed using a morphological operator *close* with a disk structuring element and then taken as input to the Fleck's [4] upgraded

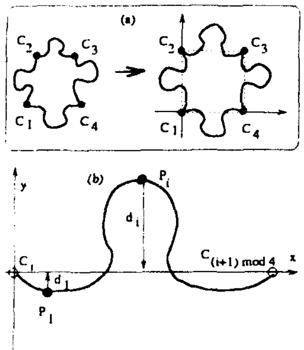

Figure 1: New boundary matching scheme.

version of Canny edge finder to detect the exterior puzzle boundary.

The algorithm detecting the four corners of a puzzle piece uses the curvatures computed for all the points on the boundary and the heuristics which are derived from the shape of a puzzle piece. For each side of a puzzle piece, bitangent and raytangent points [11] are also detected. The bitangent detection is performed by means of the Hough transform [3].

2.2. Local Matching Stage

The new boundary matching scheme uses the notion of a canonical frame [11]. The four corners on a rectangular puzzle piece are mapped into four corners of a unit square in a canonical frame(Figure 1(a)). For each canonical subcontour between C_i and $C_{(i+1) \bmod 4}$, a referential frame is associated. Given a canonical subcontour in its referential frame, let P_i be points sampled at regular intervals along the contour. Let d_i be the signed distance from P_i to the horizontal axis. We then define a mapping f, $f(i) = d_i$, and refer to the mapping f as *canonical distance function* (Figure 1(b)).

Suppose that $f_s(i)$ and $f_t(i)$ are canonical distance functions for two canonical subcontours s(slot, m sampling points) and t(tab, n sampling points) and without loss of generality that $m \geq n$. For $e = 0, \ldots,$ m-n, let

$$Cost^e_{boundary}(s,t) = \sqrt{\frac{\sum_{i=1}^{n} |f_s(i+e) - f_t(i)|^2}{n}}$$

Then the boundary matching cost between s and t is expressed as

$$Cost_{boundary}(s,t) = \text{Average}_{e \in \{0,1,\ldots,m-n\}}[Cost^e_{boundary}(s,t)]$$

The new color matching scheme starts with taking some number of sampling points P_i at regular intervals along a side and locating points P_i^{In} which are at a fixed distance from P_i toward the interior and are on the line perpendicular to the tangent line at P_i. Then, circles W_i of a radius r at P_i^{In} are data sampling windows within which color characteristics such as hue, saturation, rg/by, and RGB are extracted.

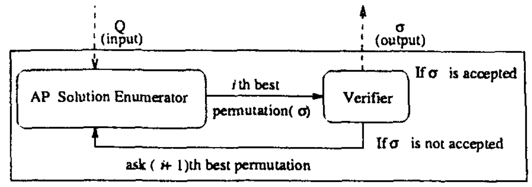

Figure 2: Block diagram of AP-based algorithm.

Assume for now that the number of sampling windows on s is same as the one on t, i.e., $m = n$. First, compute a feature histogram $H_i = \{h_1, \ldots, h_p\}$ for each circular sampling window W_i on s and t. For each pair of corresponding windows W_i^s and W_i^t, then compute a quantity u_i

$$u_i = |q_i^s - q_i^t| + \text{HistoDiff}(H_i^s, H_i^t)$$

where q_i is a median at window W_i and HistoDiff(H_i^s, H_i^t) is a measure of how much the two histograms H_i^s and H_i^t differ. More precisely, HistoDiff(H^s, H^t) is $\sum_{i=1}^{p} |h_i^s - h_i^t|$. Second, u_i are carefully examined to see whether each u_i is abnormal or not. Assuming that T_{high} and T_{low} are two thresholds with $T_{high} > T_{low}$, u_i is said to be abnormal if (a)$u_i > T_{high}$ and (b)$u_{i-1} < T_{low}$ and $u_{i+1} < T_{low}$. Finally, the color matching cost between sides s and t is computed as

$$Cost_{color}(s,t) = \sqrt{\frac{\sum_{i=1}^{n'} [u_i^{norm}]^2}{n'}}$$

where u_i^{norm} is a normal point and n' is the total number of normal points.

When m and n are different($m > n$ without loss of generality), for $e = 0, \ldots, m-n$, the quantity $Cost^e_{color}(s,t)$ can be computed in the similar manner. The color matching cost is then

$$Cost_{color}(s,t) = \text{Average}_{e \in \{0,1,\ldots,m-n\}}[Cost^e_{color}(s,t)]$$

2.3. Global Solution Stage

We give an outline of three new puzzle assembly algorithms whose detailed implementation is described in [2]. The first approach, called the *assignment problem based approach*(AP-based approach), directly applies the assignment problem to seek a global solution of the jigsaw puzzle problem. It consists of two modules, *AP solution enumerator* and *verifier*, as shown in Figure 2. The module *AP solution enumerator* generates the ith best permutation to the assignment problem with matrix Q, which is a cost matrix computed in the local matching stage. The permutation is fed to the module *verifier* to see if the permutation is a true solution to the jigsaw puzzle problem. If the permutation is found to be a solution, the verifier yields it as output. If not, the verifier requests the AP solution enumerator to issue the next best permutation.

The second approach, called the *traveling salesman problem and assignment problem based approach*(TSP&AP-based approach), is made possible by the observation that puzzle pieces can be categorized into frame pieces and interior pieces. Frame pieces, constituting the outer frame of

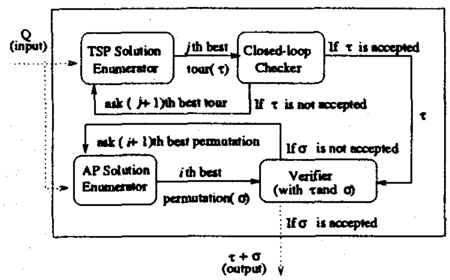

Figure 3: Block diagram of TSP&AP-based algorithm.

the completely assembled puzzle picture, have at least one straight edge. Interior pieces have no straight edge and fill the inside of the picture. Thus, TSP&AP-based approach (Figures 3) starts to assemble the frame puzzle pieces and then assemble the interior puzzle pieces.

The assembly of the frame pieces can be formulated using the traveling salesman problem. The best tour is generated and then checked to see if frame pieces represented by the tour really make a close loop. If a closed loop is made, then the tour is regarded as the solution to the assembly of the frame pieces. If not, the second best tour is generated and the same procedure is repeated until the solution is found. As soon as the solution to the frame pieces is found, the assembly of the inner pieces is initiated. The inner assembly can be achieved exactly in the same way as the AP-based approach.

Finally, the *traveling salesman problem and Kbest based approach*(TSP&Kbest-based approach) starts with assembling the frame pieces and then assembling interior pieces in the same way as the TSP&AP-based approach. The only difference between TSP&Kbest-based and TSP&AP-based approaches is that in assembling the inner pieces, the TSP&Kbest-based method instead uses a technique called *Kbest* employed by Wolfson [16]

2.4. Solution Display Stage

This stage produces a graphical representation of a completely reconstructed picture of puzzle pieces. In order to move pieces, we compute a 3×3 homogeneous transformation matrix which maps multiple equally spaced points on two sides. The algorithm for the transformation matrix iteratively seeks the best affine parameters in the first and second rows of the matrix by a least-square method, and then the best bottom two parameters in the third row by Newton's method.

3. EXPERIMENTAL RESULTS

To test the proposed algorithms, we chose six color jigsaw puzzles available on the market: *panther* (54 pieces); *car* (54 pieces); *boy* (25 pieces); *barney* (25 pieces); *elephant* (20 pieces); *bear* (12 pieces). The program was coded in Allegro Common Lisp on IBM RS/6000 workstation

Table 1 shows the results of executing three assembly algorithms over puzzle *panther* when using shape or color information separately. Table 2 supplies the results

Puzzle *panther*					
Type	Approaches	Assembly Algorithms			
		AP	TSP&AP		TSP&K
			Frame	Interior	Interior
Shape	New(B)	1000^+	24	1000^+	3
	Schwartz(S)	1000^+	1	1000^+	2
	Bitangent(R)	1000^+	1000^+	1000^+	500^+
Color	Hue	1000^+	2	1000^+	12
	Sat.	1000^+	1000^+	1000^+	500^+
	Hue&Sat.	1000^+	2	1000^+	16
	Rg&By	1000^+	13	1000^+	190
	RGB	1000^+	948	1000^+	500^+

Table 1: Assembly outcome in using shape or color information separately.

Puzzle *panther*					
Type	Color Models	Assembly Algorithms			
		AP	TSP&AP		TSP&K
			Frame	Interior	Interior
B+C	Hue	2	1	1	1
	Sat.	1000^+	4	567	2
	Hue&Sat.	1	1	1	1
	Rg&By	3	1	3	2
	RGB	1000^+	2	1000^+	2
S+C	Hue	4	1	1	1
	Sat.	1000^+	7	1000^+	4
	Hue&Sat.	2	1	1	1
	Rg&By	156	2	19	4
	RGB	1000^+	2	1000^+	3
R+C	Hue	1000^+	2	1000^+	11
	Sat.	1000^+	1000^+	1000^+	500^+
	Hue&Sat.	1000^+	2	1000^+	11
	Rg&By	1000^+	42	1000^+	62
	RGB	1000^+	1000^+	1000^+	500^+

Table 2: Assembly outcome in integrating color into shape information.

of executing three assembly algorithms when combining the boundary matching scheme and the color matching scheme. The values under TSP&K's *Interior* column are the number K required for the TSP&Kbest-based approach. In other columns, a value in each table cell means the number of iterations until the true solution to the assembly is obtained. The number n^+ means that the algorithm had not finished after n iterations (or K = n) and was stopped. The symbol B and C represents the new boundary and color matching scheme, respectively. The symbol S and R represents the Schwartz scheme[16, 12] and the bitangent canonical frame scheme [11], respectively.

As the numbers in Table 1 indicate, the three boundary matching algorithms do not show good result. However, when chromatic information is incorporated, as Table 2 shows, all three combinations B+C, S+C, and R+C gain much better results, compared with the results obtained through only the shape information. This observation implies that color information significantly aids in solving the partial boundary matching problem.

Considering that Schwartz curve matching scheme has

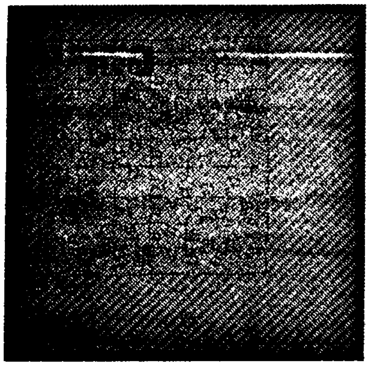

Figure 4: Reconstructed picture of 54-piece puzzle *pather*.

ever been the best in solving jigsaw puzzle problem, it is striking that the outcome of B+C is better than the outcome of S+C in all three assembly algorithms. In terms of how rapidly each assembly algorithm reaches a solution, the TSP&Kbest-based approach is the best, followed by TSP&AP-based approach, and followed by AP-based approach. The two most significant and stable color features are the Rg&By and the combined hue and saturation. RGB is the least significant and effective feature.

Due to the limitation of the space, we cannot provide the reconstructed pictures for all 6 puzzles. Figure 4 displays only the picture for 54-piece puzzle *panther*.

4. CONCLUSION

This paper proposed a new *pictorial* jigsaw puzzle solver which uses chromatic information as well as geometric shape. We have developed three new puzzle assembly algorithms and new boundary and color matching operation.

As the experimental results demonstrate, it is clear that many jigsaw puzzle problems which are hard to solve with only shape information can be easily solved when we add color information into the shape information. It is discovered that the new boundary matching scheme is comparable to the Schwartz curve matching scheme which has been, until now, the best at solving jigsaw puzzle problems. In particular, one noticeable feature about the new boundary matching scheme is that when combined with the color matching scheme, the new boundary matching scheme frequently performs better than the Schwartz scheme. We have also noticed the performance difference among three assembly algorithms. In terms of how rapidly each assembly algorithm reaches a solution, the TSP&Kbest-based algorithm is the best, followed by TSP&AP-based algorithm, and followed by AP-based algorithm.

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